Codifications of complete preorders that are compatible with Mahalanobis disconsensus measures

José Carlos Rodríguez Alcantud and Rocío de Andrés Calle and Teresa González-Arteaga

University of Salamanca, University of Salamanca, University of Valladolid

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José Carlos R. Alcantud
Universidad de Salamanca
Fac. de Economía y Empresa
Campus Unamuno
jcr@usal.es

Rocío de Andrés Calle
Universidad de Salamanca
Fac. de Economía y Empresa
Campus Unamuno
rocioac@usal.es

Teresa González-Arteaga
Universidad de Valladolid
Facultad de Ciencias
Campus Miguel Delibes
teresag@eio.uva.es

Abstract. We introduce the use of the Mahalanobis distance for the analysis of the cohesiveness of a group of linear orders or complete preorders. We prove that arbitrary codifications of the preferences are incompatible with this formulation, while affine transformations permit to compare profiles on the basis of such a proposal. This measure seems especially fit for the cases where the alternatives are correlated, e.g., committee selection when the candidates are affiliated to political parties.

Keywords: complete preorders, Mahalanobis disconsensus measure, codification.

1 Introduction

The axiomatic analysis of the measurement of the coherence in a profile of preferences has received growing attention since the seminal contribution by Bosch [5]. Some earlier analysis of that concept can be acknowledged, e.g., Hays [9] or Day and McMorris [7]. In most cases ([5], [9], or the recent Alcalde-Unzu and Vorsatz [1]) agents are presumed to linearly order the alternatives. There are proposals for extending those approaches to the case when ties are allowed, that is, when agents have complete preorders on the alternatives: see, e.g., García-Lapresta and Pérez-Román [8]. In a related line, Alcantud et al. [2] show that the case when agents have dichotomous opinions on the alternatives is both conceptually rich and technically favorable for the purpose of providing axiomatic support to the consensus indexes (as in [1] or [5]). In particular, Alcantud and Muñoz-Torrecillas [3] apply the techniques from [2] to propose indexes of sociopolitical consensus that are subsequently used to perform an empirical analysis of data from Swiss and Italian referenda.

Here we introduce the use of the Mahalanobis distance for the analysis of the cohesiveness of a group of linear orders or complete preorders. As is usual in the distance-based approaches, a codification of the preferences in order to transform profiles into matrices is needed for the purpose of building our Mahalanobis disconsensus measure. We study to what extent the choice of the codification preserves the verdict as to which profiles are more 'coherent' than others according to our proposal, which seems to be a novel question in this realm. We refer to compatibility of a class of codifications with respect to our disconsensus measure. Specifically, we show that our ranking of preorders is unique up to affine transformations of a given codification. Although we prove this for a benchmark codification, the development for the general case is completely analogous. We show by example that the choice among a number of existing alternative codifications affects the comparison between two profiles in terms of a common consensus measure (namely, our Mahalanobis disconsensus measure).

This paper is organized as follows. Section 2 is devoted to introduce basic terminology, as well as the definition of disconsensus measure. Moreover the relationship of this concept with some early approaches is included. In Section 3 we set forth the definition of our proposal of disconsensus measure, the Mahalanobis disconsensus measure and a class of complete order codifications compatible with this measure. In Section 4 a real empirical application is included. Finally, Section 5 concludes and poses questions for further research.

2 Terminology and notation

In this Section we introduce a new construct in order to compare group cohesiveness: namely, disconsensus measures. The literature usually deals with sort of a 'dual' formulation where the higher the index, the more coherence in the society's opinions. We proceed to introduce the notation for establishing our indexes and then a comparison with the standard approach is made, which produces a number of immediate examples. Advantages of our alternative construct are brought to light.

2.1 Profiles of complete preorders: definition and a canonical codification

We fix a finite set of N agents that have complete preorders (i.e., complete and transitive binary relations) on a set of k alternatives $\{x_1, ..., x_k\}$. Member i's ranking R_i of the k alternatives, with asymmetric part denoted by P_i, can be codified by numerical vectors $M_i = (m_{i1}, ..., m_{ik})$ in different forms. In the utility-oriented tradition, we request $m_{ip} \geqslant m_{iq}$ iff agent i weakly prefers alternative p over alternative q: $x_p R_i x_q$. [1] The *canonical codification* of R_i is defined by $K_i = (c_{i1}, ..., c_{ik})$ where $c_{ip} = card\{q : x_p R_i x_q\}$.[2]

Similarly, a profile $\mathcal{P}$ of complete preorders $(R_1, ..., R_N)$ is captured by an $N \times k$ real-valued matrix whose rows are $(M_1, ..., M_N)$ respectively. We write $\mathbb{M}_{N\times k}$ for the set of all $N \times k$ real-valued matrices. Thus $M \in \mathbb{M}_{N\times k}$ produces a unique profile $\mathcal{P}$ of complete preorders, although every profile of complete preorders can be associated with infinitely many matrices from $\mathbb{M}_{N\times k}$. The *canonical codification* of a profile $\mathcal{P}$ is denoted as $K_{\mathcal{P}}$.

Example 2.1 *Let R_1, R_2, R_3 be the complete preorders on $\{x_1, x_2, x_3\}$ given by*

R_1	R_2	R_3
x_1	x_3	x_2
x_2x_3	x_1x_2	x_1
		x_3

Then their respective canonical codifications are $K_1 = (3, 2, 2)$, $K_2 = (2, 2, 3)$, and $K_3 = (2, 3, 1)$. Now define $\mathcal{P}_1 = (R_1, R_2)$ and $\mathcal{P}_2 = (R_1, R_3)$. Their respective canonical codifications are

$$K_{\mathcal{P}_1} = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}, \; K_{\mathcal{P}_2} = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 1 \end{pmatrix}$$

A linear order R is an antisymmetric (xRy and yRx implies $x = y$) complete preorder.

2.2 Disconsensus measures. Relationship with earlier approaches

A *disconsensus measure* is a mapping $\delta : \mathbb{M}_{N\times k} \to [0, \infty)$ with the property:

[1] However in social choice there is a tradition that m_{ip} is the rank assigned by agent i to the p-th alternative, or the average in case of ties. This latter convention is exemplified e.g., by Black [4, p. 4] or Cook and Seiford [6, p. 623]. Both conventions can be used interchangeably.

[2] This is a standard way to prove that complete preorders on finite sets have utility representations.

i) $\delta(M) = 0$ if and only if M is unanimous (i.e., the rankings coincide for all agents).

We also deal with *normal* disconsensus measures, i.e., those that verify:

ii) *Anonymity*: $\delta(M^\sigma) = \delta(M)$ for each permutation σ of the agents and $M \in \mathbb{M}_{N\times k}$

iii) *Neutrality*: $\delta({}^\pi M) = \delta(M)$ for each permutation π of the alternatives and $M \in \mathbb{M}_{N\times k}$

Disconsensus measures enable us to compare profiles of complete preorders in terms of consensus or cohesiveness, although their use is not limited to this particular interpretation of the elements of $\mathbb{M}_{N\times k}$.

Authors like Bosch [5] and Alcalde-Unzu and Vorsatz [1] have approached the same topic by appealing to cohesiveness measures for the collective N as functions that assign to every ranking profile a real number *from the unit interval*. More importantly, in their notion the higher the assignment, the more coherence in the ranking profile. Unanimous profiles are identified by the value 1. Our approach resembles more the notion of a "measure of statistical dispersion", in the sense that 0 captures the natural notion of unanimity as total lack of variability, and then increasingly higher numbers mean more dispersion among rankings in the profile. This avoids unnecessarily cumbersome expressions due to normalizations, inversion of natural amounts by subtracting from 1, or both (as in Bosch's variety, maximum, and variance measures, or [8, Definition 8]).

Since a universally accepted definition of consensus is not available, we do a not intend to define disconsensus by opposition to consensus. They should not be taken as formal antonyms. However, consensus measures in the sense of Bosch [5, Definition 3.1] verify Anonymity and Neutrality (see also Alcantud et al. [2], Definition 1), and from a *purely technical* viewpoint, they relate to disconsensus measures as follows.

Lemma 2.2 *If μ is a consensus measure then $1-\mu$ is a normal disconsensus measure. Conversely, if δ is a normal disconsensus measure then $\frac{1}{\delta+1}$ is a consensus measure.*

Proof. We just need to recall that the mapping $i : [0,\infty) \longrightarrow (0,1]$ given by $i(x) = \frac{1}{x+1}$ is strictly decreasing.

Examples of disconsensus measures can be imported from the existing literature through Lemma 2.2.

Each disconsensus measure δ produces a ranking of profiles of complete preorders R_δ by establishing that $\mathcal{P}' R_\delta \mathcal{P}$ iff $\delta(K_{\mathcal{P}'}) \geqslant \delta(K_{\mathcal{P}})$. In words, $\mathcal{P}'$ conveys at least as much disconsensus as $\mathcal{P}$ when the disconsensus measure of the canonical codification of $\mathcal{P}'$ is at least as large as the disconsensus measure of the canonical codification of $\mathcal{P}$. As is standard, the asymmetric part of the complete preorder R_δ is denoted by P_δ.

We say that a class $\mathcal{C}$ of mappings $\mathbb{R} \longrightarrow \mathbb{R}$ is *compatible* with a disconsensus measure δ if $\delta(K_{\mathcal{P}'}) \geqslant \delta(K_{\mathcal{P}}) \Leftrightarrow \delta(f(K_{\mathcal{P}'})) \geqslant \delta(f(K_{\mathcal{P}}))$ when $f(K_{\mathcal{P}})$, $f(K_{\mathcal{P}'})$ are respective cell-by-cell transformations of $K_{\mathcal{P}}, K_{\mathcal{P}'}$ by any mapping f from $\mathcal{C}$.

Let $\mathcal{C}_s = \{f : \mathbb{R} \longrightarrow \mathbb{R}$ such that f is strictly increasing$\}$. The following technical lemma is immediate.

Lemma 2.3 *If $f \in \mathcal{C}_s$ then for each disconsensus measure (resp., normal disconsensus measure) δ, the expression $\delta_f(M) = \delta(f(M))$ for each $M \in \mathbb{M}_{N\times k}$ defines a disconsensus measure (resp., a normal disconsensus measure).*

Proof. Properties ii) and iii) are trivial. Property i) holds true because f is injective.

3 The Mahalanobis disconsensus measure

In this section we set forth the definition of our proposal of disconsensus measure based on the Mahalanobis distance between vectors, the Mahalanobis disconsensus measure. And we also identify a meaningful class of codifications of complete preorders compatible with this measure.

3.1 Definition and illustrative examples

Let $S \in \mathbb{M}_{k\times k}$, and assume that S is positive-definite. Vectors from $\mathbb{R}^k$ are row vectors. The Mahalanobis (squared) distance on $\mathbb{R}^k$ associated with S is defined by $d_S(x,y) = (x-y)S^{-1}(x-y)^t$. It is a common tool e.g., in data mining, pattern recognition, etc. We now define the *Mahalanobis disconsensus measure* on profiles on k alternatives associated with S through its associated canonical codification. The definition is as follows

$$\delta_S(\mathcal{P}) = \delta_S(K_{\mathcal{P}}) \text{ where } \delta_S(K_{\mathcal{P}}) = \frac{1}{C_N^2}\sum_{i<j} d_S(K_{R_i}, K_{R_j}) \tag{1}$$

where $C_N^2 = \frac{N(N-1)}{2}$ is the number of unordered pairs of the N agents. [3] Thus δ_S is the arithmetic average of the corresponding Mahalanobis distances between each pair of (canonically codified) individual preferences. This is in the tradition of the first definition of a consensus measure by Hays [9], as the average of Kendall's τ correlation measure between pairs of rankings. The expression (1) defines a disconsensus measure that verifies ii) but not necessarily iii).

Example 3.1 *Let S be the identity matrix. Consider the profiles of complete preorders in Example 2.1. Then $\delta_S(\mathcal{P}_1) = \delta_S(K_{\mathcal{P}_1}) = (1,0,-1)S^{-1}(1,0,-1)^t = 2$ and $\delta_S(\mathcal{P}_2) = \delta_S(K_{\mathcal{P}_2}) = (1,-1,1)S^{-1}(1,-1,1)^t = 3$ thus $\mathcal{P}_2 \, P_{\delta_S} \mathcal{P}_1$.*

Our generic specification includes some benchmark instances that derive from the following:

Example 3.2 *If S is the identity matrix (respectively, diagonal) then $\sqrt{d_S(K_{R_i}, K_{R_j})}$ is the Euclidean distance (respectively, a normalized Euclidean distance) between the canonical codification of the complete preorders R_i, R_j.*

Our choice of $d_S(x,y)$ coincides with Mahalanobis' [10] original definition. In order to exploit the inclusion of the Euclidean distance (cf., Example 3.2), some authors work with $\sqrt{d_s(x,y)}$ instead. In both cases we have distances on $\mathbb{R}^k$.

Remark 3.3 *Obviously, different specifications of Mahalanobis disconsensus measures arise if we use a codification of the profiles other than the canonical one in order to implement Equation (1). This aspect is put to practice in Example 3.7.*

3.2 A class of codifications of complete preorders compatible with the Mahalanobis disconsensus measure

In this subsection we study to which extent the choice of the numerical values that we attach to the alternatives (in order to transform preferences into vectors and profiles into matrices) changes the ranking induced by the consensus assessment from δ_S. With respect to a given codification, allowing for generic strictly increasing transformations of the values can easily change the verdict as the following simple example with the canonical codification proves:

[3] Formally, two different functions with different domains are used. We believe that our natural abusing notation does not cause any confusion henceforth.

Example 3.4 *Let S be the identity matrix. Then $\mathcal{C}_s$ is not compatible with the disconsensus measure δ_S.*

To prove it, let $R_1^{'}$, $R_2^{'}$, $R_3^{'}$ be the linear orders on $\{x_1, x_2, x_3\}$ given by $x_3 \, P_1^{'} \, x_2 \, P_1^{'} \, x_1$, $x_1 \, P_2^{'} \, x_2 \, P_2^{'} \, x_3$, $x_1 \, P_3^{'} \, x_3 \, P_3^{'} \, x_2$. Define $\mathcal{P} = (R_1^{'}, R_2^{'})$ and $\mathcal{P}^{'} = (R_1^{'}, R_3^{'})$. Their respective canonical codifications are

$$K_{\mathcal{P}} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \; K_{\mathcal{P}'} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

then $\delta_S(K_{\mathcal{P}}) = 8 > 6 = \delta_S(K_{\mathcal{P}'})$ thus $\mathcal{P} \, P_{\delta_S} \mathcal{P}^{'}$.

Now consider any $f \in \mathcal{C}_s$ such that $f(1) = 0$, $f(2) = 2$, $f(3) = 3$. We obtain

$$f(K_{\mathcal{P}}) = \begin{pmatrix} 0 & 2 & 3 \\ 3 & 2 & 0 \end{pmatrix}, \; f(K_{\mathcal{P}'}) = \begin{pmatrix} 0 & 2 & 3 \\ 3 & 0 & 2 \end{pmatrix}$$

and $\delta_S(K_{\mathcal{P}}) \geqslant \delta_S(K_{\mathcal{P}'}) \Leftrightarrow \delta_S(f(K_{\mathcal{P}})) \geqslant \delta_S(f(K_{\mathcal{P}'}))$ is false because $\delta_S(f(K_{\mathcal{P}})) = 9 < 14 = \delta_S(f(K_{\mathcal{P}'}))$.

We now proceed to identify a class of transformations of the values that are used to codify preorders, that do not dispute the ranking of the profiles in terms of Mahalanobis distances and any arbitrary chosen codification (cf., Theorem 3.5 and Remark 3.6):

Theorem 3.5 *Let $\mathcal{C}_a = \{f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = mx + b$ for some $m, b \in \mathbb{R}\}$. Then $\mathcal{C}_a$ is compatible with the disconsensus measure δ_S for each positive-definite $S \in \mathbb{M}_{k \times k}$.*

Proof. Let $\mathcal{P}$ be a profile of complete preorders $(R_1, ..., R_N)$ and $K_{\mathcal{P}}$ its corresponding canonical codification. The Mahalanobis disconsensus measure on the profile $\mathcal{P}$ associated with S is

$$\delta_S(\mathcal{P}) = \delta_S(K_{\mathcal{P}}) = \frac{1}{C_N^2} \sum_{i<j} d_S(K_{R_i}, K_{R_j})$$

or in matrix terms

$$\delta_S(K_{\mathcal{P}}) = \frac{1}{C_N^2} \sum_{i<j} \left[\left(K_{R_i} - K_{R_j}\right) S^{-1} \left(K_{R_i} - K_{R_j}\right)^t \right].$$

Fix any $f \in \mathcal{C}_a$. The Mahalanobis disconsensus measure for the profile $\mathcal{P}$ codified by the transformation of the values by f is

$$\delta_S(f(K_{\mathcal{P}})) = \frac{1}{C_N^2} \sum_{i<j} \left[\left(f(K_{R_i}) - f(K_{R_j})\right) S^{-1} \left(f(K_{R_i}) - f(K_{R_j})\right)^t \right]$$

that is

$$\delta_S(f(K_{\mathcal{P}})) = \frac{1}{C_N^2} \sum_{i<j} \left[m \left(K_{R_i} - K_{R_j}\right) S^{-1} m \left(K_{R_i} - K_{R_j}\right)^t \right] = m^2 \delta_S(\mathcal{P}).$$

This completes the proof: $\delta_S(K_{\mathcal{P}'}) \geqslant \delta_S(K_{\mathcal{P}}) \Leftrightarrow \delta_S(f(K_{\mathcal{P}'})) \geqslant \delta_S(f(K_{\mathcal{P}}))$ when $f(K_{\mathcal{P}})$, $f(K_{\mathcal{P}'})$ are respective cell-by-cell transformations of $K_{\mathcal{P}}, K_{\mathcal{P}'}$ by any mapping f from $\mathcal{C}_a$.

Remark 3.6 *To avoid misleading statements we have referred Theorem 3.5 to the canonical codification. As is apparent, the precise form of the codified vectors K_{R_i} plays no role in the proof. Thus if a Mahalanobis disconsensus measure is defined with a different codification of the preorders, compatibility of $\mathcal{C}_a$ with such an alternative formulation is assured too.*

Nevertheless Remark 3.6 does not mean that the choice of the codification procedure is irrelevant. Generically speaking, the procedures to attach consensus indexes that rely on codifications of the orderings are crucially shaped by the precise specification of such coding. For our tool of analysis, the following situation exemplifies the relevance of that auxiliary first step (that permits to use a fixed consensus measure):

Example 3.7 *Let S be the identity matrix. Consider the profiles of complete preorders in Example 2.1. If we use the Mahalanobis consensus measure as has been defined (i.e., in terms of canonical codifications) then Example 3.1 shows $\delta_S(K_{\mathcal{P}}) = 2 < 3 = \delta_S(K_{\mathcal{P}})$ therefore $\mathcal{P}' \, P_{\delta_S} \mathcal{P}$.*

If we use codifications of the complete preorders in the vein of Cook and Seiford [6] then each order R_i must be linearized and each alternative p receives the average of the canonical codifications c_{iq} of all alternatives q that are indifferent to p.[4] Therefore we need to refer our computations to

$$K'_{\mathcal{P}} = \begin{pmatrix} 3 & 1.5 & 1.5 \\ 1.5 & 1.5 & 3 \end{pmatrix}, \; K'_{\mathcal{P}'} = \begin{pmatrix} 3 & 1.5 & 1.5 \\ 2 & 3 & 1 \end{pmatrix}$$

and now the Mahalanobis disconsensus measure δ'_S that arises prescribes $\delta'_S(K'_{\mathcal{P}}) = 4.5 > 3.5 = \delta'_S(K'_{\mathcal{P}'})$ thus $\mathcal{P} \, P_{\delta'_S} \mathcal{P}'$.

As is apparent, the technical decision about the codification procedure alters the conclusion of which of the two profiles is more tight in terms of coherence as measured by a fixed Mahalanobis disconsensus measure.

Similar negative examples can be designed for the aforementioned related proposals in prior literature.

4 A real empirical application: Ranking universities in the world

In order to put in practice our disconsensus measure proposal we develop a real empirical example. We build on The Academic Ranking of World Universities (ARWU). [5] In this yearly report various rankings on universities worldwide are established by a group of researchers at the Center for World-Class Universities of Shanghai Jiao Tong University. They also developed the Academic Ranking of World Universities by Broad Subject Fields (ARWU-FIELD) and by Subject Fields (ARWU-SUBJECT). We proceed to check which of these two latter rankings is more tight in terms of coherence, according to our Mahalanobis disconsensus measure associated with the identity matrix. For simplicity, in our application we consider ARWU-FIELD and ARWU-SUBJECT 2012 rankings on five top universities, namely, x_1 =Berkeley, x_2 =Harvard, x_3 =MIT, x_4 =Princeton, and x_5 =Stanford. [6]

According to the ARWU-SUBJECT 2012:

R_1	R_2	R_3	R_4	R_5
Princeton	Harvard	Harvard	Stanford	Harvard
Harvard	MIT	Berkeley	MIT	MIT
Berkeley	Berkeley	Stanford	Berkeley	Berkeley
Stanford	Princeton	MIT	Princeton	Stanford
MIT	Stanford	Princeton	Harvard	Princeton

Here the R_i represent the rankings for Mathematics, Physics, Chemistry, Computer Sciences, and Economics/Business. Their respective canonical codifications are $K_1 = (3, 4, 1, 5, 2)$,

[4] Linearization can be done in various ways, but this fact does not modify the codification. We stress that our variant is forced by the fact that we use different conventions for higher codes, as discussed in Footnote 1.

[5] Source: http://www.shanghairanking.com/aboutus.html

[6] The rankings have been retrieved from http://www.shanghairanking.com/index.html

$K_2 = (3,5,4,2,1)$, $K_3 = (4,5,2,1,3)$, $K_4 = (3,1,4,2,5)$, and $K_5 = (3,5,4,1,2)$. We now define $\mathcal{P} = (R_1, R_2, R_3, R_4, R_5)$, then its canonical codification is

$$K_{\mathcal{P}} = \begin{pmatrix} 3 & 4 & 1 & 5 & 2 \\ 3 & 5 & 4 & 2 & 1 \\ 4 & 5 & 2 & 1 & 3 \\ 3 & 1 & 4 & 2 & 5 \\ 3 & 5 & 4 & 1 & 2 \end{pmatrix}.$$

When S equals the identity matrix, $\delta_S(K_{\mathcal{P}}) = \frac{1}{10} \cdot 204 = 20.4$.

According to the ARWU-FIELD 2012:

R_1'	R_2'	R_3'	R_4'	R_5'
Harvard	MIT	Harvard	Harvard	Harvard
Berkeley	Stanford	MIT	Stanford	Berkeley
Princeton	Berkeley	Stanford	Berkeley	MIT
MIT	Princeton	Berkeley	Princeton, MIT	Princeton
Stanford	Harvard	Princeton		Stanford

Here the R_i' represent the rankings for Natural Sciences and Mathematics, Engineering/Technology and Computer Sciences, in Life and Agriculture Sciences, Clinical Medicine and Pharmacy, and Social Sciences. [7] Their respective canonical codifications are $K_1' = (4,5,2,3,1)$, $K_2' = (3,1,5,2,4)$, $K_3' = (2,5,4,1,3)$, $K_4' = (3,5,2,2,4)$, and $K_5' = (4,5,3,2,1)$. We now define $\mathcal{P}' = (R_1', R_2', R_3', R_4', R_5')$, then its canonical codification is

$$K_{\mathcal{P}'} = \begin{pmatrix} 4 & 5 & 2 & 3 & 1 \\ 3 & 1 & 5 & 2 & 4 \\ 2 & 5 & 4 & 1 & 3 \\ 3 & 5 & 2 & 2 & 4 \\ 4 & 5 & 3 & 2 & 1 \end{pmatrix}.$$

When S equals the identity matrix, $\delta_S(K_{\mathcal{P}'}) = \frac{1}{10} \cdot 168 = 16.8$.

As a conclusion, the profile of rankings provided for the ARWU-FIELD 2012 is more coherent than the corresponding profile for the ARWU-SUBJECT 2012.

5 Conclusion and future research

In this paper we explore the problem of measuring the degree of disconsensus in a setting where experts express their opinions on alternatives by means of linear or complete preorders. To that purpose we use the general concept of disconsensus measure. In this initial contribution we define a notion of compatibility with disconsensus measures. We also introduce the particular Mahalanobis disconsensus measure based on the Mahalanobis (squared) distance of numerical vectors. As to results, we firstly prove that arbitrary preference codifications are incompatible with our proposal. Secondly, we demonstrate that the verdict of the Mahalanobis disconsensus measure is preserved under linear transformation functions, which seems to be a novel conclusion in this realm. Finally, our proposal has been illustrated with a real empirical example about different profiles of rankings provided for a reduced set of top universities.

The Mahalanobis distance is a powerful tool in Statistics and its applications. It allows to introduce corrections due to correlations among the variables. In our case, this could be used

[7] Princeton and MIT are not mentioned among the list of institutions classified by the R_4' criterion.

to adjust the basic Euclidean approach (as applied in Section 4) to account for the effect of correlated rankings. This will be the subject of future research.

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